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Static Analysis of Composite Plates by a Simple Theory Quasi 3-D

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ABSTRACT/RESUME

Abstract: *This paper presents a static analysis of laminated composites plates by employing a novel higher-order shear deformation theory with stretching effect (quasi-3D HSDT). This theory accounts for both transverse shear and normal strains effects by a parabolic variation of all displacements through the thickness and satisfies the stress-free boundary conditions on the top and bottom surfaces of the plate without using shear correction factor. The displacement field of the proposed theory has only five unknowns, which is even less than the other existing high-order shear deformation plate theories. The principle of virtual works is used to derive the governing equations and boundary conditions. The closed form solutions are obtained by using Navier procedure for cross-ply laminated composite plates subjected to sinusoidal load for simply supported boundary conditions. The numerical results are compared with those predicted by other theories to show the effects of shear deformation and thickness stretching on displacement and stresses.*

I. Introduction

Fiber reinforced composite are widely employed in various engineering industries such as the aerospace, automotive, marine and other structural applications due to superior mechanical properties of these materials. In the past three decades, investigations on laminated composite plates have attracted considerable attention, and a variety of laminated theories has been developed. Such as the classical plate theory (CPT), the first-order shear deformation theory (FSDT) and the higher-order shear deformation plate theories (HSDT). Various researches have proposed a number of HSDTs to predict accurately the bending, buckling and vibration responses of laminated composite plates. Soldatos [1] proposed hyperbolic shear deformation theory for the flexure analysis of laminated composite plates. An analytical solution is presented by Kant and Swaminathan [2] for the bending analysis of laminated composite and sandwich plates using a higher order refined theory. Akavci [3] developed a novel hyperbolic theory in terms of tangent and secant functions for the analysis of plates. Sayyad and Ghugal [4] proposed a trigonometric shear deformation theory taking into account transverse shear deformation effect as well as transverse normal strain effect or static flexure of cross-ply laminated composite plates. A general mathematical formulation in Green–Lagrange sense for geometrically nonlinear free vibration response of laminated composite plates has been developed by Swain *et al.* [5] based on the higher-order polynomial shear deformation theory. Katariya *et al.* [6] used an HSDT kinematic model to solve the bending and vibration problem of the skew sandwich composite plate with laminate facing and isotropic and orthotropic core for different geometrical parameters. An exact series solution for free vibration of a rectangular orthotropic plate with two opposite edges rotationally restrained and free edges was obtained by Zhang and Zhang [7] using the finite integral transform method. Sahla *et al.* [8] applied a simple four-variable trigonometric shear deformation model with undetermined integral terms for the free vibration analysis of simply supported antisymmetric laminated composite and soft core sandwich plates. Recently, Sayyad and Ghugal [9] presented a generalized higher-order shell theory for the static and free vibration analyses of laminated composite and sandwich spherical shells.

In the present paper, an analytical solution of the static analysis of laminated composite plates subjected to sinusoidal distributed loads is proposed by using a simple quasi-3D HSDT. Just five independent unknowns are employed in the present theory against six independent unknowns or more independent unknowns employed in the corresponding shear and normal deformations theories. The stretching thickness effect is more important to assess the effect of local stress concentration due to concentrated load. This local effect can be successfully evaluated by the present method. The performance of the present formulation is confirmed by comparing the results obtained with classical plate theory (CPT), first- order shear deformation theory (FSDT) [10], high- order shear deformation theory (HSDT) of Reddy [11], quasi-3D TSDT generated by Sayyad and Ghugal [4] and the exact elasticity solution provided by Pagano [12] whenever applicable.

II. Materials and Methods

II.1. Theoretical Formulation

Consider as a numerical example of laminated composite plate with total thickness h, made up of n orthotropic layers with the coordinate system as shown in Figure 1.

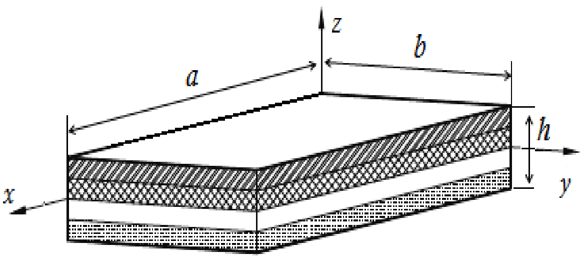

Figure 1. *Coordinate system used for a typical laminated plate.*

II.2. The Displacement Field

In the present theory this displacement at any point of coordinates (x, y, z) in the plate includes a transverse displacement (w) which is composed of three components, namely: bending, shear and stretching and can be written as:

$$
\begin{aligned}
u(x,y,z) &= u_0(x,y) - z\frac{\partial w_b}{\partial x} - \left[\frac{5z^3}{3h^2} - \frac{z}{4}\right]\frac{\partial w_s}{\partial x} \\
v(x,y,z) &= v_0(x,y) - z\frac{\partial w_b}{\partial y} - \left[\frac{5z^3}{3h^2} - \frac{z}{4}\right]\frac{\partial w_s}{\partial y} \qquad (1) \\
w(x,y,z) &= w_b(x,y) + w_s(x,y) + g(z)\,\varphi(x,y)
\end{aligned}
$$

II.3. Strain Displacement Relations

The non-zero strains associated with the displacement field in Equation. (1) are:

$$
\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix} + z\begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_b}{\partial y^2} \\ -2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix} + f(z)\begin{Bmatrix} -\dfrac{\partial^2 w_s}{\partial x^2} \\ -\dfrac{\partial^2 w_s}{\partial y^2} \\ -2\dfrac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix},
$$

$$
\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \dfrac{\partial w_s}{\partial y} + \dfrac{\partial \varphi}{\partial y} \\ \dfrac{\partial w_s}{\partial x} + \dfrac{\partial \varphi}{\partial x} \end{Bmatrix}, \ \varepsilon_z = g'(z)\varphi \qquad (2)
$$

Where: $f(z) = z\left(\dfrac{5z^2}{3h^2} - \dfrac{1}{4}\right)$ and $g(z) = 1 - f'(z)$ (3)

II.4. Constitutive Relations

The laminate is generally made of several orthotropic layers. Each layer of the laminated plate is assumed to be in a state of three-dimensional stress so that the constitutive relationships for any layer (*k*) in the coordinate system are written as follows:

$$
\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}^{(k)} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{13} & 0 & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{23} & 0 & 0 & 0 \\ \bar{Q}_{13} & \bar{Q}_{23} & \bar{Q}_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{Q}_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{Q}_{66} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}^{(k)} \qquad (4)
$$

where $\bar{Q}_{ij}^{k}$ are the transformed stiffness constants, which report the elastic behavior of each layer to the laminate reference axis system of the laminates.

II.5. Governing Equations

The principle of virtual work is used to study the static flexure problem of the laminated plates under consideration. The static equilibrium equations associated with the present simple quasi-3D theory are obtained in the form:

$$
\begin{aligned}
&\delta u_0: \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \quad \delta v_0: \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\
&\delta w_b: \ \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + q = 0 \\
&\delta w_s: \ \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} + q = 0, \qquad (5) \\
&\delta \varphi: \ \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} - N_z = 0
\end{aligned}
$$

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Where $N_x, N_y, N_{xy}, M_x^b, M_y^b, M_{xy}^b, M_x^s, M_y^s, M_{xy}^s S_{xz}^s, S_{yz}^s$ and N_z are the stress resultants.

III. Numerical Results and Discussion

In this section, some numerical examples are listed in tables 1 and 2 to verify the effectiveness of the present formulation in predicting the static response of simply supported antisymmetric cross-ply laminated plate. The following lamina properties are employed [12]:

$$E_1 = 25E_2,\ E_3 = E_2,\ G_{12} = G_{13} = 0.5E_2,\ G_{23} = 0.2E_2$$
and $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$ (6)

In addition, the following dimensionless displacements and stresses have been employed throughout the tables and figures:

$$\bar{w}\left(\frac{a}{2},\frac{b}{2},0\right) = \frac{100h^3E_3}{qa^4}w,$$

$$\left(\bar{\sigma}_x,\bar{\sigma}_y\right)\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right) = \frac{h^2}{qa^2}\left(\sigma_x,\sigma_y\right), \qquad (7)$$

$$\bar{\tau}_{xy}\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right) = \frac{h^2}{qa^2}\tau_{xy},\ \bar{\tau}_{xz}\left(0,\frac{b}{2},0\right) = \frac{h}{qa}\tau_{xz},$$

$$\bar{\tau}_{yz}\left(\frac{b}{2},0,0\right) = \frac{h}{qa}\tau_{yz}$$

TABLE 1. *Comparison of transverse displacement for simply supported two-layer (0/90) square laminated plate subjected to sinusoidal load.*

a/h	**Theory**	$\bar{w}$
	Present	1.9652
	Sayyad and Ghugal (TSDT)	1.9424
4	Reddy (HSDT)	1.9985
	Mindlin (FSDT)	1.9682
	Kirchhoff (CPT)	1.0636
	Pagano (Exact)	2.0670
	Present	1.2123
	Sayyad and Ghugal (TSDT)	1.2089
10	Reddy (HSDT)	1.2161
	Mindlin (FSDT)	1.2083
	Kirchhoff (CPT)	1.0636
	Pagano (Exact)	1.2250

TABLE 2. *Comparison of stresses for simply supported two-layer (0/90) square laminated plate subjected to sinusoidal load.*

a/h	**Theory**	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
	Present	0.8982	0.0968	0.0562	0.3132	0.3132
4	Sayyad and Ghugal (TSDT)	0.9063	0.0964	0.0562	0.3189	0.3189
	Reddy (HSDT)	0.9060	0.0891	0.0577	0.3128	0.3128
	Mindlin (FSDT)	0.7157	0.0843	0.0525	0.2274	0.2274
	Kirchhoff (CPT)	0.7157	0.0843	0.0525	---	---
	Pagano (Exact)	0.8410	0.1090	0.0591	0.3210	0.3130
	Present	0.7446	0.0865	0.0530	0.3191	0.3191
110	Sayyad and Ghugal (TSDT)	0.7471	0.0876	0.0530	0.3261	0.3261
	Reddy (HSDT)	0.7468	0.0851	0.0533	0.3190	0.3190
	Mindlin (FSDT)	0.7157	0.0843	0.0525	0.2274	0.2274
	Kirchhoff (CPT)	0.7157	0.0843	0.0525	---	---
	Pagano (Exact)	0.7302	0.0886	0.0535	0.3310	0.3310

A simply supported cross ply (0/90) square laminate under sinusoidal transverse load is examined. The comparison of results of transverse displacement and stresses for slenderness ratios 4 and 10 is demonstrated in Tables 1 and 2. The maximum deflections predicted by present model are in good agreement with other solutions of Sayyad and Ghugal [4] for (0/90) cross-ply laminated plate whereas CPT underestimates the results for all slenderness ratios. The axial normal stress $\bar{\sigma}_x$ determined by the present formulation is in excellent agreement with that of Sayyad and Ghugal [4] for all slenderness ratios. Both the present theory and the theory proposed by Sayyad and Ghugal [4], give the same values of the shear stress $\bar{\tau}_{xy}$ and transverse shear stress $\bar{\tau}_{xz}$ (see Figures 2 and 3). The proposed theory predicts more accurate transverse shear stresses than those provided by other refined theories as compared to exact values.

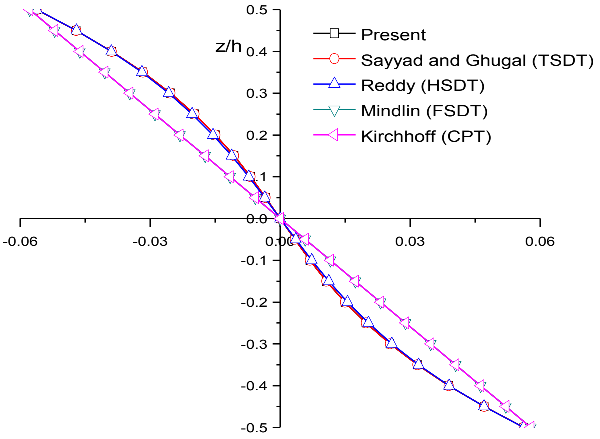

Figure 2.*Through thickness distribution of the shear stress* $\bar{\tau}_{xy}$ *of (0/90) cross-ply laminated plate under sinusoidal loading for a/h =4.*

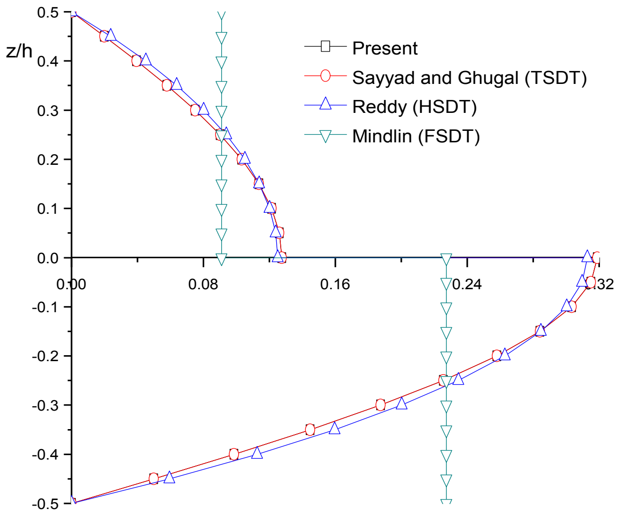

Figure 3. *Through thickness distribution of the transverse shear stress* $\bar{\tau}_{xz}$ *of (0/90) cross-ply laminated plate under sinusoidal loading for a/h =4.*

IV. Conclusion

This paper presents a flexural analysis of antisymmetric laminated composite plates by using a simple quasi-3D parabolic theory under sinusoidal transverse load. The governing equations are obtained by using the principle of virtual works. The results show that this theory is efficient in predicting more accurate results than the FSDT and other HSDTs with higher number of unknown variables.

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